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Technical Report No. 32-604

Constants and Related Data for Use in Trajectory Calculations

As Adopted by the Ad Hoc NASA Standard Constants Committee

Victor C. Clarke, Jr.

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CALIFORNIA INSTITUTE OF TECHNOLOGY
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Systems Analysis

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ABSTRACT

The establishment of a standard set of constants and ephemeris data for trajectory and orbit determination computations is essential to ensure accuracy and consistency between the various NASA agencies and contractors participating in the space program. One of the conditions for correct comparison and use of numerical results is that the same set of constants and ephemeris data be used. Further, in the selection of numerical values for the constants, recent measurements should be considered so that the values are near optimum, i.e., the best available. Periodic review of the values is necessary to incorporate significant improvements. However, too-frequent changes are undesirable.

The establishment, selection, and review of a NASA Standard Set of Constants for use in trajectory and orbit calculations has been the concern of several NASA engineers, namely, R. K. Squires of Goddard Space Flight Center, E. A. Braunlich, formerly of Marshall Space Flight Center, Paul Brumberg of Manned Spacecraft Center, V. C. Clarke, Jr., the author of the present report, and other engineers within NASA. In late 1960, plans were made to form an ad hoc group to adopt a NASA Standard Set of Constants. As a result of this activity, a set of constants was adopted in 1961 and reviewed, with minor changes, in 1963. Periodic review on about a two-year basis is planned.

This report gives the adopted values of key constants for use in trajectory calculations, information regarding the procurement of best available ephemeris data, and other work of the ad hoc group.

I. INTRODUCTION

The purpose of this report is to set forth the list of constants and related data for use in space trajectory computation in NASA programs. In the past, efforts have been made by trajectory design personnel within NASA to standardize a set of constants. Such efforts resulted in two meetings. The first meeting was held at Marshall Space Flight Center on May 4, 1961, and was attended by representatives of Goddard and Marshall Space Flight Centers, the Jet Propulsion Laboratory, Langley and Lewis Research Centers, and what was then the Space Task Group (now the Manned Spacecraft Center). The minutes of this meeting were published by R. K. Squires of GSFC and contain the values and sources of the data that were agreed on as a tentative

NASA standard set of constants. Although this set of standards has never been officially sanctioned or published, it has been adopted and used by JPL and by those contractors associated with the *Ranger, Mariner,* and *Surveyor* programs, which are administered by JPL. Again, to the best of the author's knowledge, this set of constants is being used by GSFC and MSC. Thus these constants have been in widespread use while not being officially sanctioned.

The second meeting, chaired by W. Kaula of GSFC and the author, was held at Goddard Space Flight Center on May 16, 1963. The proceedings of this meeting have not yet been published, but a summary memorandum of the results was issued by E. D. Geissler of MSFC on May 20, 1963. The meeting resulted in agreement on several points:

1. The astronomical and geodetic constants proposed in a document entitled *Natural Environment and Physical Standards for Project Apollo*[a] were adopted. These constants are almost identical with the constants presented in the original version of the present report (Ref. 1). Two exceptions were made to the Project *Apollo* document:

    a. The Earth–Moon mass ratio is to have the value 81.3015 as opposed to the 81.35 used in the Project *Apollo* document. This improved value is based on data obtained from the flight of *Mariner* 2 (Ref. 2).

    b. The radius of the Earth for use in the geopotential function was changed from 6378166 meters to 6378165 meters. This slight change was made in order to agree with the Earth model established by W. Kaula at the May 1961 NASA Committee meeting and reaffirmed at the May 1963 meeting.

2. The geometric model of the Earth to be used is the World Geodetic System, also known as the Astrogeodetic World Datum (Ref. 3).

3. Data to be published by JPL under the title "Ephemeris Tape System" will be used for the ephemerides. Details of the content of this system are given in Section V of this report.

4. A committee chaired by W. Kaula of GSFC was formed to study the determination and standardization of tracking station positions. Recommendations of this committee are given in an appendix.

For those interested in adopting this set of constants, it should be noted that their chief qualification is standardization. However, a real attempt was made to select a set which might be termed the "best available at the time." The plain truth, especially with regard to lunar and planetary constants, is that there is a fair degree of uncertainty as to their values, and different experimenters have determined values of the same constant which sometimes differ widely. Fortunately, a fair activity is at present being stimulated within NASA to make new and better determinations of these constants. One such activity was the recent precise determination (Ref. 4) of the astronomical unit by JPL using the Goldstone Tracking Station. The results of that determination are reflected below in that the value given is based on the data reduction as of May 31, 1961. The reported (and more final) value as of March 8, 1962, is 1 au = 149,598,845 $\pm$250 km, which is only slightly different from the "standardized" value given below.

In the area of lunar and planetary ephemerides, an intense effort is being conducted at JPL by P. Peabody, D. Holdridge, and N. Block, in cooperation with the Nautical Almanac Office, to improve the quality of the ephemerides. Already significant improvements in both position and velocity ephemerides have been made by these workers, as discussed in Section V.

[a]Issued by the Office of Manned Space Flight, preliminary document M-DE-8020.008.

II. EARTH CONSTANTS

A. Potential Function

The potential function of the Earth is expressed by

$$\Phi(R,\phi) = \frac{GM_E}{R}\left[1 + \frac{J_2 R_E^2}{2R^2}(1 - 3\sin^2\phi) + \frac{J_3}{2}\frac{R_E^3}{R^3}(3 - 5\sin^2\phi)\sin\phi - \frac{J_4}{8}\frac{R_E^4}{R^4}(3 - 30\sin^2\phi + 35\sin^4\phi)\right]$$

or by

$$\Phi(R,\phi) = \frac{GM_E}{R}\left[1 + \frac{JR_E^2}{3R^2}(1 - 3\sin^2\phi) + \frac{HR_E^3}{5R^3}(3 - 5\sin^2\phi)\sin\phi + \frac{D}{35}\frac{R_E^4}{R^4}(3 - 30\sin^2\phi + 35\sin^4\phi)\right]$$

where (Ref. 5)

$$GM_E = 3.986032\ (\pm 0.000030) \times 10^5\ \text{km}^3/\text{sec}^2$$

$$R_E = \text{equatorial Earth radius}$$

$$= 6378.165\ (\pm 0.025)\ \text{km}$$

$$R = \text{geocentric radius}$$

$$\phi = \text{geocentric latitude}$$

$$J_2 = 1082.30\ (\pm 0.2) \times 10^{-6}$$

$$J_3 = -2.3\ (\pm 0.1) \times 10^{-6}$$

$$J_4 = -1.8\ (\pm 0.2) \times 10^{-6}$$

For the J, D, H form of the potential, we have

$$J = \frac{3}{2}J_2 = 1.62345\ (\pm 0.00030) \times 10^{-3}$$

$$H = \frac{5}{2}J_3 = -0.575\ (\pm 0.025) \times 10^{-5}$$

$$D = -\frac{35}{8}J_4 = 0.7875\ (\pm 0.0875) \times 10^{-5}$$

The values of GM_E, J, and R_E are consistent with the values of the geodetic parameters of flattening f and gravity g_e,

$$f = 1/298.30$$

$$g_e = 0.00978030\ \text{km/sec}^2$$

The values of R_E, f, and g_e are consistent with those specified in the Department of Defense (DOD) World Geodetic System 1960.

B. Geometric Form

The geometric form of the Earth is the Astrogeodetic World Datum of Irene Fischer (Ref. 3).

$$f = \text{flattening}$$

$$= 1/298.3$$

$$R_E = \text{equatorial Earth radius}$$

$$= 6378.166\ \text{km}$$

C. Rotation Rate

The Earth's rotation rate ω_E is given by

$$\omega_E = \frac{360}{86164.09892 + 0.00164T}\ \text{deg/sec}$$

where T is the number of Julian centuries of 365.25 days from 1900 Jan 0.5 UT (Julian date = 2415020.0). For the year 1963,

$$\omega_E = 4.1780742 \times 10^{-3}\ \text{deg/sec}$$

D. Atmospheric Model

The reference atmosphere for boost vehicle trajectory calculations for launches from Cape Kennedy, Florida, is given in Refs. 6, 7, and 8. Intensive studies of this subject have been conducted by O. E. Smith of MSFC.

E. Miscellaneous Constants

Miscellaneous constants are defined as follows:

1. Conversion Factors. The National Bureau of Standards chart reproduced in Fig. 1 gives the necessary conversion factors between the English and metric systems.

2. Velocity of Light. The velocity of light c (Ref. 9) in a vacuum is based on the determination of Froome:

$$c = 299792.5 \text{ km/sec}$$

3. Ephemeris–Universal Time Reduction. The relation between Ephemeris Time (ET) and Universal Time (UT) is given in Ref. 10. For 1963, the difference is

$$\Delta T = ET - UT = 35 \text{ sec}$$

III. LUNAR CONSTANTS

A. Potential Function

The potential function of the Moon is expressed by

$$\Phi(R, \mu, \lambda) = \frac{GM_m}{R}\left[1 + \frac{A + B + C - 3I}{2R^2 M_m}\right]$$

or by

$$\Phi(R, \mu, \lambda) = \frac{GM_m}{R}\left[1 + J\left(\frac{R_m}{R}\right)^2\left(\frac{1}{3} - \sin^2\mu\right) + L\left(\frac{R_m}{R}\right)^2 \cos^2\mu \cos 2\lambda\right]$$

where

R = selenocentric radius

μ = selenographic latitude

λ = selenographic longitude

and where

Earth–Moon mass ratio (Ref. 2)

$$M_E/M_m = 81.3015 \ (\pm 0.0033)$$

$$GM_m = 4.9027779 \times 10^3 \text{ km}^3/\text{sec}^2$$

Mean lunar radius $R_m = 1738.09$ km

Moments of inertia about principal axis

$$A = 0.88781798 \times 10^{35} \text{ kg m}^2$$

$$B = 0.88800195 \times 10^{35} \text{ kg m}^2$$

$$C = 0.88836978 \times 10^{35} \text{ kg m}^2$$

which values were calculated from the values $a = 1738.57$ km, $b = 1738.21$ km, $c = 1737.49$ km, taken from Ref. 11 using

$$A = \frac{M_m}{5}(b^2 + c^2)$$

$$B = \frac{M_m}{5}(c^2 + a^2)$$

$$C = \frac{M_m}{5}(a^2 + b^2)$$

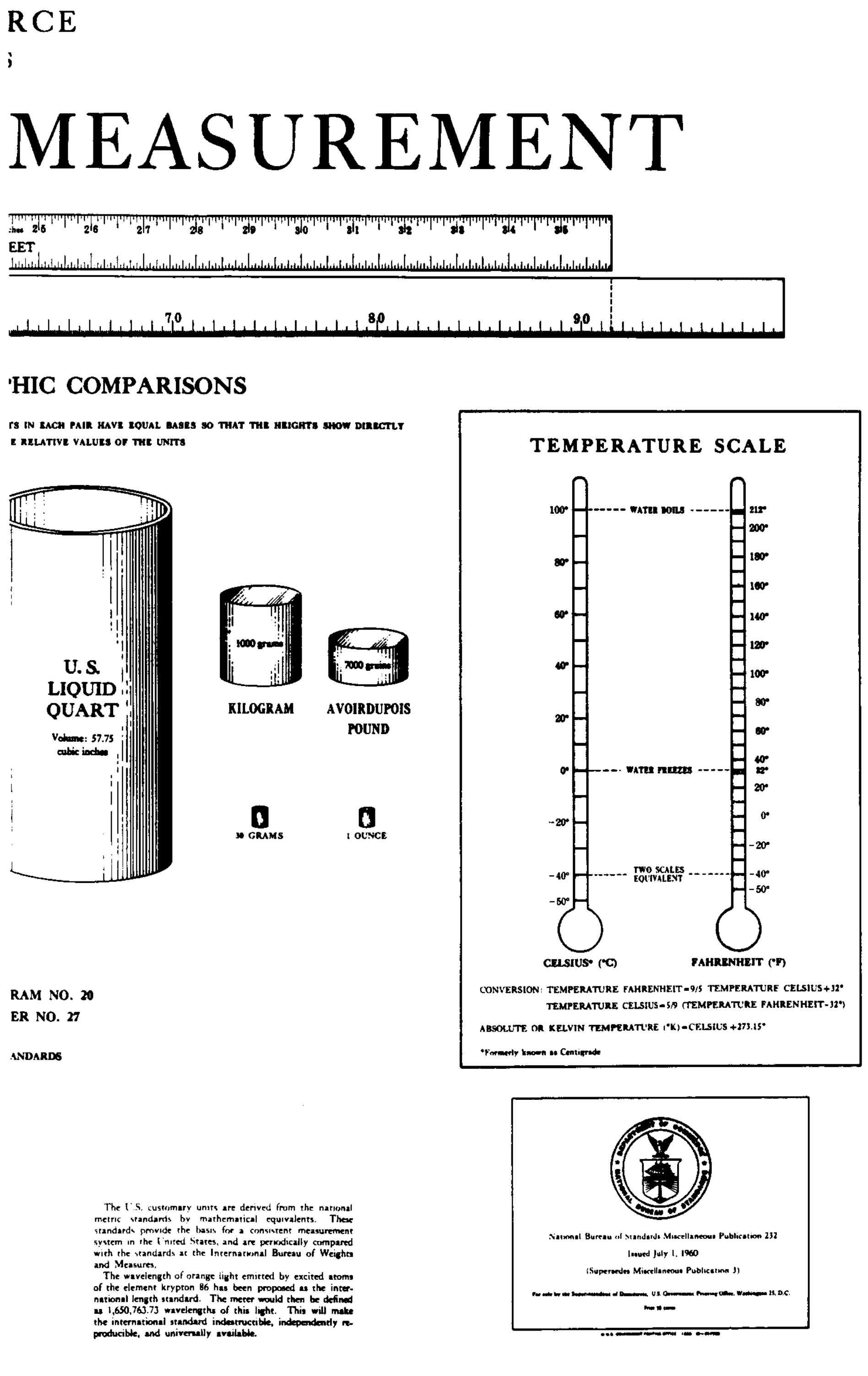

Fig. 1. The metric system of measurement

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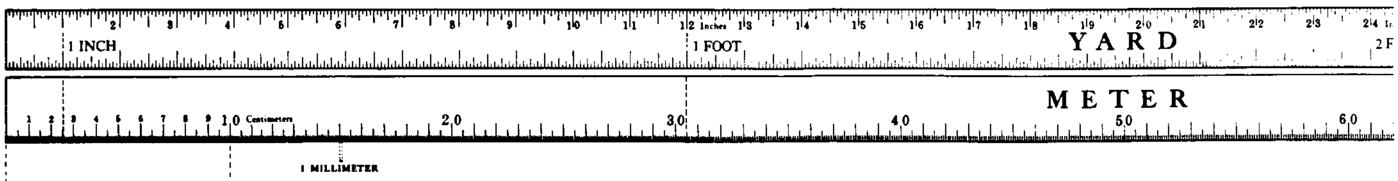

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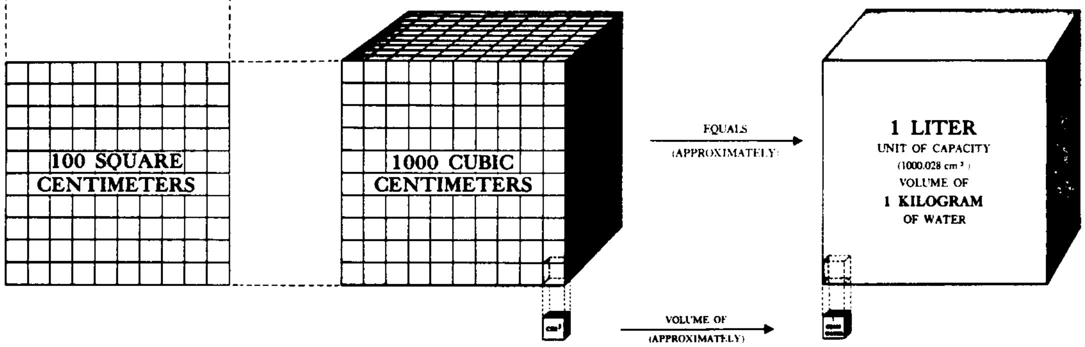

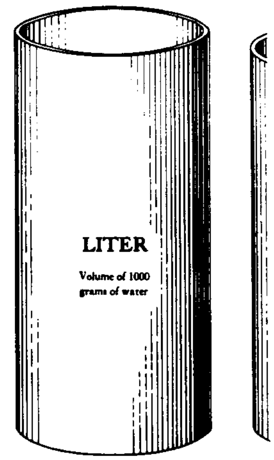

THE METRIC SYSTEM

The metric system comprises the meter as the unit of length and the kilogram as the unit of mass. The unit of fluid capacity, the liter, is the volume of one kilogram of water and is approximately equal to 1/1000 of a cubic meter. All other metric units are the decimal subdivisions or multiples of these three basic units, and are named according to the table of prefixes below.

In conjunction with the metric system the Celsius temperature scale is used and the second is taken as the unit of time. The temperature scale is defined by certain fixed points (see table). The second is defined as $\frac{1}{31{,}556{,}925.9747}$ of the tropical year 1900 (approximately 1/86,400 of the average day) and is determined by astronomical observation.

Responsibility for the preservation and promotion of the metric system is centered at the International Bureau of Weights and Measures, at Sèvres, France, which was established under the terms of the Treaty of the Meter of 1875. The United States is an adherent to the Treaty of the Meter.

Multiples and Submultiples	Prefixes	Symbols
1 000 000 000 000 = 10^{12}	tera	T
1 000 000 000 = 10^{9}	giga	G
1 000 000 = 10^{6}	mega	M
1 000 = 10^{3}	kilo	k
100 = 10^{2}	hecto	h
10 = 10	deka	dk
0.1 = 10^{-1}	deci	d
0.01 = 10^{-2}	centi	c
0.001 = 10^{-3}	milli	m
0.000 001 = 10^{-6}	micro	μ*
0.000 000 001 = 10^{-9}	nano	n
0.000 000 000 001 = 10^{-12}	pico	p

For example, 1000 meters (or 10^3 meters) is called a kilometer, and one millionth of a gram (or 10^{-6} gram) is called a microgram.

*1 millionth of a meter is called a micron, and is abbreviated simply μ

Most Common Uses of Prefixes

Length

1 000 millimeters mm, 100 centimeters cm, 10 decimeters dm	= 1 METER (about 40 inches)	m
1 000 meters	= 1 kilometer (about 5/8 mile)	km

Mass

1 000 000 micrograms μg, 1 000 milligrams mg	= 1 GRAM (about 15 grains)	g
1 000 grams	= 1 kilogram (about 2 pounds)	kg

Capacity

1 000 milliliters ml	= 1 LITER (about 1 quart)	l
1 000 liters	= 1 kiloliter	kl

EQUIVALENTS

1 YARD = 0.9144 METER

1 AVOIRDUPOIS POUND = 0.453 592 37 KILOGRAM

(See Federal Register, July 1, 1959)

Length

Centimeter	= 0.3937 inch
Meter	= 3.28 feet
Meter	= 1.094 yards
Kilometer	= 0.621 statute mile
Kilometer	= 0.5400 nautical mile
Inch	**= 2.54 centimeters**
Foot	**= 0.3048 meter**
Yard	**= 0.9144 meter**
Statute mile	= 1.61 kilometers
Nautical mile	**= 1.852 kilometers**

Area

Sq centimeter	= 0.155 sq inch
Sq meter	= 10.76 sq feet
Sq meter	= 1.196 sq yards
Hectare	= 2.47 acres
Sq kilometer	= 0.386 sq mile
Sq inch	= 6.45 sq centimeters
Sq foot	= 0.0929 sq meter
Sq yard	= 0.836 sq meter
Acre	= 0.405 hectare
Sq mile	= 2.59 sq kilometers

Volume

Cu centimeter	= 0.0610 cu inch
Cu meter	= 35.3 cu feet
Cu meter	= 1.308 cu yards
Cu inch	= 16.39 cu centimeters
Cu foot	= 0.0283 cu meter
Cu yard	= 0.765 cu meter

Capacity

Milliliter	= 0.0338 U.S. fluid ounce
Liter	= 1.057 U.S. liq quarts
Liter	= 0.908 U.S. dry quart
U.S. fluid ounce	= 29.57 milliliters
U.S. liq quart	= 0.946 liter
U.S. dry quart	= 1.101 liters

Mass or Weight

Gram	= 15.43 grains
Gram	= 0.0353 avdp ounce
Kilogram	= 2.205 avdp pounds
Metric ton	= 1.102 short or net tons
Grain	= 0.0648 gram
Avdp ounce	= 28.35 grams
Avdp pound	= 0.4536 kilogram
Short or net ton	= 0.907 metric ton

Equivalents shown in bold face type are exact; all others are approximate
For complete conversion tables, see NBS Miscellaneous Publication 233

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where

$$M_m = \frac{M_E}{81.3015}$$

and

$$M_E = 5.975 \times 10^{27} \text{ gm}$$

and where

The constants J and L are defined by

$$J = \frac{3}{4}\left(\frac{2C-A-B}{R_m^2 M_m}\right) = \frac{3}{20}\left(\frac{a^2+b^2-2c^2}{R_m^2}\right)$$

$$= 0.00031066$$

$$L = \frac{3}{4}\left(\frac{B-A}{R_m^2 M_m}\right) = \frac{3}{20}\left(\frac{a^2-b^2}{R_m^2}\right)$$

$$= 0.000062148$$

B. Selenographic Coordinates

The formulas and expressions for computing the transformations to selenographic coordinates are given in Ref. 12. It should be noted, however, that the formulas for the nutation in longitude and obliquity given in this reference do not apply. Better expressions for these quantities are given by Woolard (Ref. 13).

C. Earth Radii/Kilometer Conversion Factor for the Lunar Ephemeris

It is important to note (Ref. 14) that the value (in km) of the Earth radius to be used for converting the Lunar Ephemeris from Earth radii to kilometers is *not* the actual radius of the Earth as given in Section I. Rather, an artificial value, b, must be used which is calculated from the formula

$$b = 86.315745\,(GM_E + GM_m)^{1/3}$$

$$= 6378.3255 \text{ km}$$

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IV. PLANETARY CONSTANTS

A. Astronomical Unit

The astronomical unit adopted is

$$1 \text{ au} = 149.599000 \times 10^6 \text{ km}$$

B. Mass Ratios and Gravitational Constants

The mass ratios and gravitational constants are given in Table 1. The gravitational constants were computed using the Gaussian constant and the Sun-to-planet mass ratios from

$$GM_{planet} = \frac{GM_{sun}}{M_s/M_{planet}}$$

Table 1. Planetary mass ratios and gravitational constants

Planet	Mass ratio M_s/M_{planet}	Gravitational constant GM_p, au^3/day^2	Gravitational constant GM_p, km^3/sec^2	Reference
Sun	1	$2.959122083 \times 10^{-4}$	$1.32715445 \times 10^{11}$	
Mercury	6120000	4.835167×10^{-11}	2.168553×10^{4}	Rabe, 1950 (Ref. 15)
Venus	408539.5	$7.2431725 \times 10^{-10}$	3.2485340×10^{5}	Anderson, Null, 1963 (Ref. 2)
Earth	332951.3	8.887552×10^{-10}	3.986032×10^{5}	Kaula, 1961 (adopted)
Mars	3088000	9.582649×10^{-11}	4.297780×10^{4}	Clemence, Brouwer, 1955 (Ref. 16)
Jupiter	1047.39	2.825234×10^{-7}	1.267106×10^{8}	Clemence, Brouwer, 1955 (Ref. 16)
Saturn	3500	8.454635×10^{-8}	3.791870×10^{7}	Baker, 1961 (adopted)
Uranus	22869	1.293945×10^{-8}	5.803292×10^{6}	Clemence, Brouwer, 1957
Neptune	18889	1.566585×10^{-8}	7.026072×10^{6}	Biesbroeck, 1957
Pluto	400000	7.397895×10^{-10}	3.317886×10^{5}	Rabe

V. LUNAR AND PLANETARY EPHEMERIDES*

A. Ephemeris Tape System

A new set of magnetic tapes carrying ephemeris data in the form of positions and velocities of the planets and the Moon and the nutations in longitude and obliquity has been developed at JPL, superseding the ephemeris tapes described in Ref. 1, Part V.

However, the data contained in these tapes cannot be considered final in view of the many current investigations into the theories of the motion of the planets and the Moon. Accordingly, the procedures, computer programs, and archives of ephemeris data, as preserved on other magnetic tapes which were used in developing the new ephemeris tapes, have been collected into an operational system which will permit issuing updated ephemerides whenever such a new issue is desirable and

*This section was written by P. Peabody of the JPL Computer Applications and Data Systems Section.

feasible. This system is called the JPL Ephemeris Tape System. The tapes developed under this system are called JPL Ephemeris Tapes.

Special attention has been given to the problems of identifying and documenting all aspects of ephemeris data acquisition and processing and insuring the accuracy thereof. The system is described in detail in Ref. 17. Its principal features are the following:

1. Ephemeris data is acquired or generated from the best available sources. In most cases this procedure has required new evaluations of basic source theories, with the application of corrections wherever they are known. These new evaluations have been preserved on magnetic tapes and constitute the source tape library of the ephemeris system. The current contents of the source tape library are described in Section V-B.
2. Velocity as well as position data is provided, where this data is obtained from a numerical integration of the equations of motion, with orbital parameters chosen so as to give the least squares fit to source positions. Thus, position and velocity are gravitationally interconsistent. The orbit fitting program together with some results obtained is described in Section V-C. Fitted position-velocity data has been generated for all bodies except Mercury and the Moon. Velocities for these bodies are also provided for the sake of compatibility and have been derived by numerical differentiation.
3. Complete data is provided over the time interval 1950 to 2000 for all planets and the Moon. The data is thus more extensive than probably required for any particular problem. On the other hand, it is no longer necessary to generate special ephemeris tapes for special jobs. Actually, it is possible to extend the interval over which the ephemeris data is provided to times prior to 1950 and after 2000. In fact, some of the source theory evaluations have already been so extended.

Items available to users of the JPL Ephemeris Tapes will be the report defining their usage (Ref. 17), an IBM 7094 subroutine EPHEM for interpolating in the tapes, and a sequence of sets of JPL Ephemeris Tapes, each set having its own identifying document. Each set comprises three tapes collectively spanning the years 1950–2000, with overlaps of data between tapes. The tape format is described in Ref. 17 as well as in each ephemeris tape document. The data is presented in IBM 7094 double-precision floating point, except for nutations, which are in single precision.

The contents of the first set of JPL Ephemeris Tapes are defined in Table 2. The second set is obtained from the first by replacing the positions and velocities of all bodies except Mercury and the Moon by fitted data. The first set provided the necessary connection to previously published ephemerides. The second set will be the standard in use at JPL for some time.

Table 2. Contents of JPL Ephemeris Tapes

First set

Data	Step	Reference
Lunar positions, nutations	0.5-day	Brown's Improved Lunar Theory, DP 7094 evaluation by N. Block
Mercury positions	2-day	Newcomb's Theory with Clemence's Corrections, DP 7094 evaluation by N. Block
Venus positions	4-day	Newcomb's Theory with Duncombe's Corrections, DP 7094 evaluation by N. Block
Earth–Moon barycenter positions	4-day	Newcomb's Theory with Duncombe's Corrections, DP 7094 evaluation by N. Block
Mars positions	4-day	Hansen-Clemence Provisional Theory, cards generated from SP programs at USNO
Outer planets	4-day	SSEC numerical integration, transcribed to Themis tape at Lawrence Radiation Laboratory, edited at JPL, with corrections for inner planets using DP 7094 program developed by N. Block

Note: All velocities and nutation rates obtained by numerical differentiation. DP = double-precision; SP = single precision.

Second set

Lunar positions, velocities; nutations and rates; Mercury positions and velocities identical to first set.

All other bodies with position and velocity taken from numerical integration least squares fit to the source positions of JPL Ephemeris Tapes, Set 1, over 50-year arcs.

B. Source Position and Nutation Data

Input to the JPL Ephemeris System consists of predictions of the positions of the planets and the Moon and the nutations in longitude and obliquity. The positions of the planets are referred to the heliocentric rectangular coordinate system of the mean equator and equinox of 1950.0 = JD 2433282.423 and measured in au. The positions of the Moon are referred to the geocentric coordinate system of the same equator and equinox and measured in units commonly called "Earth radii."

The lunar positions and the nutations were obtained via an IBM 7094 computer program, developed by N. Block of JPL, which evaluates the Brown Improved Lunar Theory (not the tables) in double-precision. The program description and tabulations of the data are given in Ref. 18. This evaluation will constitute the source of the data published by the Nautical Almanac Office of the British Naval Observatory as the official lunar position predictions.

The coordinates of Mercury, Venus, and the Earth–Moon barycenter were obtained via IBM 7094 computer programs, also developed by N. Block, which evaluate the Newcomb theories in double-precision. The programs are described in Ref. 19. These programs admit the application of corrections to the mean elements in the form $A + Bt$. The various source tapes so far generated are:

Mercury: Newcomb's Theory, no corrections (Ref. 20). Newcomb's Theory, with corrections deduced by G. Clemence (Ref. 21).

Venus: Newcomb's Theory, no corrections (Ref. 22). Newcomb's Theory, with corrections deduced by R. Duncombe (Ref. 23).

Earth–Moon Barycenter: Newcomb's Theory, no corrections (Ref. 24). Newcomb's Theory, with corrections deduced by R. Duncombe (Ref. 23). Newcomb's Theory, with the correction of $4''.78t$ to the mean longitude as applied by P. Herget (Ref. 24).

In addition to these new evaluations, which are the preferred source position data, the data carried on the Themis tapes, which were developed by the Lawrence Radiation Laboratory and used in previous development of the STL–JPL Ephemeris Tapes and the Ephemerides for the JPL Space Trajectories Program as described in Ref. 1, has been edited and preserved in the source tape library.

The coordinates of Mars were obtained on cards from the U. S. Naval Observatory as evaluations of Clemence's third-order Hansen theory, with provisional values of the elements (Ref. 25).

The coordinates of the outer planets were taken from the Themis tapes and were obtained originally as the SSEC numerical integration of the motion of the five outer planets (Ref. 26). These coordinates are preserved in the source tape library. In addition, the corrections worked out by Clemence in order to refer the coordinates to heliocentric (Ref. 27) were applied by an IBM 7094 computer program developed by N. Block and also preserved. The latter data is the source data preferred for the JPL Ephemeris Tapes.

C. *Fitted Position and Velocity*

The procedures for fitting the position–velocity ephemeris obtained from a numerical integration to source position predictions were developed according to the experience acquired in generating the Venus and Earth–Moon barycenter fits used in the JPL Space Trajectories Program ephemeris and described in Ref. 28. These procedures were incorporated into an IBM 7094 computer program (PLOD) by J. Devine of JPL.

The equations of motion are integrated in a heliocentric equatorial reference frame, using Cowell's method in summed form, and retaining eighth differences. All computations are carried to double-precision. In all cases the range of integration includes many sidereal periods of the planets, and the integration step size is chosen so as to insure 12-place accuracy in the calculated positions.

The fitting procedure is a standard regression scheme which calculates and applies differential corrections to orbital elements until no further significant decrease in the sums of squared residuals can be obtained.

The orbital elements are taken to be the osculating elliptic ecliptic Keplerian elements of the starting epoch, except that argument of perihelion is replaced by longitude of perihelion to avoid difficulty with zero inclination, as is encountered in fitting the Earth–Moon barycenter. This choice of parameters for defining the motion gives better separation than does position and velocity at epoch or equatorial elliptic elements.

The residuals are taken in ecliptic latitude and longitude so as to remove the apparent inconsistency between the scale and the period of the source theories, as indicated by the periodic components in the residuals in latitude and longitude as plotted in Ref. 28. These periodic components are, of course, still present in the radius vector and therefore in the residuals in rectangular coordinates.

REFERENCES

1. Clarke, V. C., Jr., *Constants and Related Data Used in Trajectory Calculations at the Jet Propulsion Laboratory,* Technical Report 32-273, Jet Propulsion Laboratory, Pasadena, California, May 1, 1962.

2. Anderson, J. D., and Null, G. W., "The Evaluation of Certain Physical Constants from the Radio Tracking of Mariner II," AIAA Preprint 63-424, paper presented at the AIAA Astrodynamics Conference, August 19–21, 1963, Yale University, New Haven, Connecticut.

3. Fischer, I., "An Astrogeodetic World Datum from Geoidal Heights Based on the Flattening $f = 1/298.3$," *Journal of Geophysical Research,* Vol. 65, No. 7, July 1960, pp. 2067–2076.

4. Muhleman, D. O., Holdridge, D. B., and Block, N., *The Astronomical Unit Determined by Radar Reflections from Venus,* Technical Report 32-221, Jet Propulsion Laboratory, Pasadena, California, March 18, 1962.

5. Kaula, W. M., "A Review of Geodetic Parameters," paper presented at the International Astronomical Union Symposium No. 21, "The System of Astronomical Constants," Paris, May 27–31, 1963. Published as Goddard Space Flight Center Document X-640-63-55.

6. Smith, O. E., *A Reference Atmosphere for Patrick Air Force Base, Florida,* NASA TN D-555, Washington, D. C., July 29, 1960.

7. Smith, O. E., *A Reference Atmosphere for Patrick Air Force Base, Florida,* NASA TN D-595, Washington, D. C., March 1961.

8. Smith, O. E., and Chenoweth, H. B., *Range of Density Variability from Surface to 120 km Altitude,* NASA TN D-612, Washington, D. C., July 1961.

9. Cohen, E. R., Crowe, K. M., and Du Mond, J. W. M., *Fundamental Constants of Physics,* Vol. 1, Interscience Publishers, New York, 1957.

10. *American Ephemeris and Nautical Almanac,* published annually by the U. S. Government Printing Office, Washington, D. C.

11. Makemson, M. W., Baker, R. M. L., Jr., and Westrom, G. B., "Analysis and Standardization of Astrodynamic Constants," *Journal of Astronautical Sciences,* Vol. 8, No. 1, Spring 1961.

12. Kalensher, B. E., *Selenographic Coordinates,* Technical Report 32-41, Jet Propulsion Laboratory, Pasadena, California, February 24, 1961.

13. Woolard, E. W., "Theory of the Rotation of the Earth Around Its Center of Mass," *Astronomical Papers of the American Ephemeris,* Vol. XV, Pt 1, U. S. Government Printing Office, Washington, D. C., 1953.

14. Clarke, V. C., Jr., "Earth Radii/Kilometer Conversion Factor for the Lunar Ephemeris," *AIAA Journal,* Vol. 2, No. 2, 1964, pp. 363–364.

15. Rabe, E. K., "Derivation of Fundamental Astronomical Constants from the Observations of Eros During 1926–1945," *Astronomical Journal,* Vol. 55, 1950, pp. 112–125.

16. Clemence, G. M., and Brouwer, D., "The Accuracy of the Coordinates of the Five Outer Planets and the Invariable Plane," *Astronomical Journal,* May 1955, p. 118.

REFERENCES (Cont'd)

17. Peabody, P., Scott, J. F., and Orozco, E. G., *Users' Description of Jet Propulsion Laboratory Ephemeris Tapes*, Technical Report 32-580, Jet Propulsion Laboratory, Pasadena, California, March 2, 1964.

18. Block, N., *IBM 7094 Evaluation of the Brown Improved Lunar Theory*, Jet Propulsion Laboratory, Pasadena, California (to be published).

19. Block, N., *IBM 7094 Evaluation of Newcomb's Theories of Mercury, Venus, and Earth–Moon Barycenter, with Arbitrary Corrections to the Mean Elements*, Jet Propulsion Laboratory, Pasadena, California (to be published).

20. Newcomb, S., "Tables of the Heliocentric Motion of Mercury," *Astronomical Papers of the American Ephemeris*, Vol. VI, Pt. II, U. S. Government Printing Office, Washington, D. C., 1898.

21. Clemence, G. M., "The Motion of Mercury, 1765–1937," *Astronomical Papers of the American Ephemeris*, Vol. XI, Pt. I, U. S. Government Printing Office, Washington, D. C., 1943.

22. "Tables of the Heliocentric Motion of Venus," *Astronomical Papers of the American Ephemeris*, Vol. VI, Pt. III, U. S. Government Printing Office, Washington, D. C., 1898.

23. Duncombe, R. L., "Motion of Venus," *Astronomical Papers of the American Ephemeris*, Vol. XVI, Pt. I, U. S. Government Printing Office, Washington, D. C., 1958.

24. "Tables of the Motion of the Earth on Its Axis and Around the Sun," *Astronomical Papers of the American Ephemeris*, Vol. VI, Pt. I, U.S. Government Printing Office, Washington, D. C., 1898.

25. "Theory of Mars-Completion," *Astronomical Papers of the American Ephemeris*, Vol. XVI, Pt. II, U.S. Government Printing Office, Washington, D. C., 1962.

26. Eckert, W. J., Brouwer, D., and Clemence, G. M., "Coordinates of the Five Outer Planets, 1653–2060," *Astronomical Papers of the American Ephemeris*, Vol. XII, U.S. Government Printing Office, Washington, D. C., 1951.

27. Clemence, G. M., "Perturbations of the Five Outer Planets by the Four Inner Ones," *Astronomical Papers of the American Ephemeris*, Vol. XIII, Pt. V, U.S. Government Printing Office, Washington, D. C., 1954.

28. Peabody, P., and Block, N., "Planetary Position-Velocity Ephemerides Obtained by Special Perturbations," *AIAA Journal*, Vol. 1, No. 12, pp. 2812–2815, December 1963.

APPENDIX

Standards for Geodetic Location of Tracking Stations

The following material was originally presented as a memorandum from W. M. Kaula (GSFC) to the NASA Earth and Lunar Model Committee at the Conference on Astronomical and Geodetic Constants held at GSFC on May 16, 1963.

MEMORANDUM DATE: June 20, 1963

TO: NASA Earth and Lunar Model Committee

FROM: W. M. Kaula

SUBJECT: Standards for Geodetic Location of Tracking Stations

1. Reference:

Memo, MSFC, "Conference on Astronomical and Geodetic Constants (Earth and Lunar Models) held at GSFC on May 16, 1963," dtd. May 20, 1963, Par. 2.

2. Purposes:

a. To recommend standards for the measurement, computation, and documentation of tracking station positions.
b. To furnish guidance as to the means of accomplishing geodetic location.
c. To recommend the information regarding tracking station position which should be provided to users of tracking data.
d. To furnish guidance to evaluate accuracy of geodetic positions.

3. Survey of Tracking Station Positions:

a. Information as to the below listed items should always be sought from the Director, U.S. Coast and Geodetic Survey, Washington 25, D.C., for stations within the U.S., and from the Commanding Officer, U.S. Army Map Service, Washington 25, D.C., for stations outside the U.S.

(1) Location and description of the nearest geodetic control points, horizontal and vertical, to the tracking station.

(2) The recommended method to obtain the position of the tracking station with respect to the nearest geodetic control points within ±5 meters.

(3) The recommended agencies to accomplish the survey, or to furnish further information.

b. If the tracking station is of high accuracy (i.e., directions within ±3", or ranges within ±25 meters, or velocities within ±10 cm/sec), it is worthwhile to connect it by triangulation or traverse to geodetic control

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points up to 150 miles away. The normal method of survey connection--i. e., overland for distances less than a few tens of miles--will be a two-way traverse using a tellurometer for the distances and a Wild T-2 theodolite for horizontal and vertical angles. In addition, an astronomic azimuth should be observed at the station with the theodolite.

c. If the tracking station is not within 150 miles of geodetic control, an astronomic position and azimuth should be observed with a T-2. Such locations will normally be near the sea; the elevation above sea level should be obtained by leveling or carefully controlled barometric altimetry. If the location is a small island, astronomic positions should be observed on opposite sides, connections made by traverse to the tracking stations, and the mean of the two positions thus obtained taken as the station's position.

4. Computation of Tracking Station Position:

A standard method prescribed in U. S. C. and G. S. and A. M. S. manuals should be used to calculate latitude and longitude (geodetic or astronomic) and elevation above sea level. If the geoid height is calculated for a station connected to a geodetic control system, the height must be taken from a geoid map calculated from astro-geodetic data (not gravimetric) on the same datum and ellipsoid as was used to calculate the station latitude and longitude. (For a station position based on astronomic observations, a gravimetric geoid height may be used.)

Rectangular coordinates (u, v, w) referred to axes in the direction (0°, 0°), (0°, 90°E), (90°N) should be calculated by:

a_e = semimajor axis of reference ellipsoid
f = flattening of reference ellipsoid
ϕ = geodetic latitude
λ = longitude
h = elevation above sea level
N = geoid height

$$e^2 = 2f - f^2 \tag{1}$$

$$\nu = a_e(1 - e^2 \sin^2\phi)^{-1/2} \tag{2}$$

$$u = (\nu + h + N) \cos\phi \cos\lambda \tag{3}$$

$$v = -(\nu + h + N) \cos\phi \sin\lambda \tag{4}$$

$$w = \left[(1 - e^2)\, \nu + h + N)\right] \sin\phi \tag{5}$$

- 2 -

Datum shifts are preferably expressed as translations in rectangular coordinates Δu, Δv, Δw. This manner of expression eliminates the ambiguity which can occur in expressing a datum shift in geodetic coordinates concurrent with a change of reference ellipsoid dimensions (i.e., given $\Delta\phi$, $\Delta\lambda$, ΔN, Δa_e, Δf, do the $\Delta\phi$, $\Delta\lambda$, ΔN reflect only the change due to Δu, Δv, Δw, or do they also include the effect of Δa_e, Δf?)

The relationships between corrections amounting to shifts of less than 5 km can be expressed quite accurately by simple differentials of equations (1) through (5).

Rectangular coordinates for station positions based on astronomic observations should be calculated using an a_e of 6378165 (or 6) meters and an f of 1/298.30.

5. <u>Documentation of Tracking Station Position:</u>

a. The following information should be provided in the station position description. The enclosed example "SECOR SITE NEW HAVEN, INDIANA" (Encl 1) includes items (1)-(9), (11)-(13) on the list:

(1) Date records received at office of record.
(2) General description of survey.
(3) Instrumentation and method of observation.
(4) All misclosures and other indicators of accuracy.
(5) Datum and ellipsoid on which computations were performed.
(6) Location of description, computations, and field records.
(7) A sketch map of the survey.
(8) Geodetic latitude and longitude (ϕ, λ) of station.
(9) Name and date of datum; if nonstandard, plus reference to publication or other source.
(10) Semimajor axis and flattening of ellipsoid.
(11) Elevation above sea level, h.
(12) Sketch and verbal description of station location.
(13) Azimuths and distances to adjacent control stations.
(14) Geoid height (N), if available.
(15) Description of geoid height calculation, including full reference to any publication used.

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(16) Astronomic latitude and longitude of the station, or any other directions used for referring altitude-and-azimuth or direction cosine measurements.

(17) Rectangular coordinates (u, v, w) of the station, based on items (8), (9), (10), (11), and (14) above.

(18) Shifts (Δu, Δv, Δw) to other datums, if available, including the names of the other datums and full reference to the publications or documents from which the shifts were obtained or a description of how the shifts were calculated.

(19) Number of the station in any system in which it has been numbered, such as the COSPAR system (see COSPAR Information Bull. No. 10, Aug. 1962, and also SAO Spec. Rep. No. 69, July 1961), or the AFCRL system (see AFCRL Operational Plan for Project ANNA, September 1962).

b. Remarks

(1) If uncertainties have been estimated for any of the coordinates or shifts in items (8), (14), (17), or (18) above, they may be included in the station description provided that there is a careful statement of what the uncertainties are--i. e., with respect to the nearest geodetic control (usually on the order of ±2 meters), or to the origin of the geodetic datum (from ±5 to ±25 meters), or to the Earth's center of mass (from ±25 to ±300 meters).

(2) Range and range rate observations should always be corrected to refer to the fixed point to which the coordinates (8), (14), (17), or (18) apply; if the electronic measurement is made to a point which moves around appreciably (as, e. g., on the large Goldstone dish), this correction should be applied by the observing agency and not left to the user.

(3) It should always be clearly designated what reference axes are used to refer published altitude-and-azimuth or direction cosine measurements: the astronomic latitude and longitude or the geodetic latitude and longitudes.

(4) The number of the station in item (19) above should be changed if the station is shifted more than 5 meters.

- 4 -

(5) It is desirable that all stations be numbered in the largest system thereof extant--i.e., the COSPAR system. This system is maintained by Dr. B. G. Pressey, Radio Research Center, Slough, England. Part I, Optical Stations, was published in 1962; Part II, Radio Stations, will be published shortly.

(6) In any list or other abbreviated description of tracking stations in any system, the minimum information provided should include items (6), (8), (9), (10), (11), (14), (17), and (19) of the full description. Item (18) should also be given if the station position is not referred to a standard geodetic datum (e.g., NAD 1927).

6. Evaluation of Tracking Station Position Accuracy.

a. The remarks on pages 2-6 of Enclosure B to the Minutes of the NASA Earth Model meeting May 4, 1961 (Encl 2) still apply. In particular, the remarks at the top of page 4 apply to the manner of location of the Goldstone radar: being in a mountainous area, its astronomic position would have an uncertainty on the order of ±350 meters, and the device of matching slopes with the geoid map would at best remove the ±170 meters "continental" part, leaving about

$$\sqrt{350^2 - 170^2} = \pm 300 \text{ meters}$$

b. Indications of datum position accuracy obtained from satellite tracking. The following shifts from NASA 702 positions were obtained from Baker-Nunn camera observations of five satellites ranging from 33 to 96 deg in inclination:

		Shift	σ(NASA TN 702)
		Meters	Meters
Americas	Δu	-24	±26
	Δv	-33	22
	Δw	-2	22
Europe-Africa-Siberia-India	Δu	+37	23
	Δv	-57	29
	Δw	+12	23
Japan-Korea-Manchuria	Δu	-57	40
	Δv	+60	53
	Δw	+10	40
Australia	Δu	-110	75
	Δv	+33	90
	Δw	+67	35
Argentina	Δu	+244	180
	Δv	-15	160
	Δw	+36	160
Hawaii	Δu	-26	140
	Δv	+59	230
	Δw	-290	±230

The σ's suggested in Enclosure 2 thus seem to be only slightly on the low side.

The uncertainties of the shifts obtained from satellites indicated by the scatter of solutions from different orbits ranged from ±4 to ±23 meters; more realistic values based on comparison of the geometric geoid heights resulting and the gravitational geoid heights would be ±30 meters for the major triangulation systems and ±60 meters for the isolated stations. Accuracy of connection between stations within geodetic datums was checked by solutions in which the stations in South America and South Africa were assumed to be on different datums than the stations in the northern hemisphere connected to the same geodetic system. The South American station shifted 24 meters and the South African station shifted 14 meters with respect to the northern hemisphere datums.

c. Definition of a datum. There seems to be some confusion between two separate matters, geodetic datum and ellipsoid specifications. We propose as the definition of a datum:

> "A location with respect to a geocentric coordinate origin of a set of points fixed to the Earth and all located relative to each other by geodetic triangulation, trilateration, and traverse."

The location can be expressed in any sort of coordinates--rectangular, cylindrical, spherical, or "geodetic": latitude, longitude, and height referred to an ellipsoid of specified dimensions. In all orbit computing systems I know of, the coordinate origin is defined as the Earth's center of mass, either explicitly or implicitly, by making the geodetic-to-inertial transformation as a pure rotation and by not having a first-degree harmonic in the geopotential expression. The significance of specifying that the relationship in position between the points constituting the system be through triangulation, trilateration, and traverse (rather than through gravimetry or astronomy) is that thereby the errors in relative position should be appreciably less than their common error in position with respect to the Earth's center of mass. Hence for applications where the accuracy involved is intermediate in magnitude between these two errors, all stations on the same datum can be considered as translating together.

Of course, stations not connected by triangulation, trilateration, and traverse should never be considered as being put on the same datum merely by referring their positions to ellipsoids of the same dimension.

On the other hand, there often will exist cases where stations which are connected by geodetic control do not have their positions referred to the same datum. If the difference in position between the two datums is not stated by a reliable source in an unequivocal form such as Δu, Δv, Δw, it is advisable to consult the Department of Geodesy, Army Map Service. Such consultation is particularly advisable if the stations are more than about 2000 km apart. Geodetic control is generally calculated in two dimensions, neglecting the scale correction required by the difference between geoid and ellipsoid. Datum shifts covering limited areas are generally made in a two-dimensional manner. The neglect of the third dimension makes little difference for distances less than about 2000 km. At greater distances, the errors due to this neglect will mount rapidly, so that the shape of the geoid must be calculated from deflections of the vertical and the appropriate "Molodenskii corrections" applied to the calculated positions.

7. Comments on Published Listing of Tracking Station Positions:

a. SAO Spec. Pub. 59, 1961: The Position of the Baker-Nunn Camera Stations. The description of the datum transformations applied is correct and unambiguous, and should be understood after referral to p. 101 of Smithsonian Contributions to Astrophysics (Vol. 3, no. 9). However, the description is incomplete in that it fails to give the coordinates of the datum origins or the specifications of the reference ellipsoids, so the reader cannot duplicate the computations. Also, the results are far from the best attainable because the ellipsoid and the deflections of the vertical used are obsolete. The list of station positions is defective in that the datums are not specified. Items (1), (2), (3), (4), (6), (7), (12), (13), (16) of the tracking station description in Par. 5a above are not given, nor is it stated whether they are available.

b. JHU-APL SRA-179, 1962: Transit Program Station Locations. Stations which are not connected by geodetic survey are listed as being on the same datum (e.g., Australia and England); it is pointless to say that they are on the same datum with an accuracy of ±800 meters, since a better accuracy with reference to the Earth's center of mass can often be attained. The definition of the coordinate accuracies given is not clear. The specifications of the reference ellipsoids are not given. The method and the numerical parameters involved in the shift to the APL reference system are not given. Items (1), (2), (3), (4), (5), (6), (7), (10), (12), (13), (15), (16), and (18) of the tracking station description in Par. 5a above are not given, nor is it stated whether they are available.

c. NASA-OMSF Prog. Dir. M-DE 8020.008, 1963: Natural Environment and Physical Standards for Project Apollo. The information on page 31 is correct and unambiguous. It perhaps might be emphasized that "a geocentric coordinate system" is not a united datum. A description of the method of datum transformation, the parameters involved, and estimates of accuracy are entirely lacking. The spherical coordinates given are adequate in place of rectangular coordinates, but the station list lacks the geodetic latitude and longitude, elevation, datum and a number for each station, and the address from which further detailed information--items (1) through (16) and (18) of Par. 5a--can be obtained.

8. Recommended Representation on Tracking Station Committee:

NASA HQ; Goddard SFC; JPL; Manned SC, Houston; Marshall SFC; Smithsonian AO; STL; Aerospace; APL; USNWL, Dahlgren; ACIC, St. Louis; AFCRL; USC and GS; Army Map Service.

W. M. Kaula

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Enclosure 1

SECOR SITE
NEW HAVEN, INDIANA
P.O. 64123-001
29 Nov 1961

The following is a description and evaluation of a survey for the Secor Site at New Haven, Indiana.

Field Records for establishing permanently marked horizontal and vertical control stations at New Haven, Indiana were received at the Americas Division 15 November 1961.

The survey consisted of a tellurometer traverse originating on Second Order USC&GS triangulation station "DAWKINS" 1947, running west to "CASAD WEST" AMS 1961 and looping back to Dawkins USC&GS 1947. From "Casad West" tellurometer lengths were measured to "Casad" AMS 1961 and "Casad East" AMS 1961.

Horizontal Angles and Zenith Distances were observed with a Wild T-2 Theodolite. Angular closure of the 4 station traverse was 00".04. Tellurometer Lengths consisted of 12 fine readings and a check measurement from an off-set station. The lengths appear to have 2nd order Traverse Accuracy.

The starting azimuth for the traverse was "DAWKINS" to "DAWKINS AZIMUTH MARK". It is assumed that this published azimuth is within 1 second of the true Geodetic Azimuth. An angular check from the azimuth mark to a triangulation station was not made. The traverse closure ratio was 1:44,940 with a linear error of 0.295 meters. The resulting position of "Casad West" will probably be within 0.3 meters in relation to the second order control in the area. The small triangle formed by "Casad West", "Casad" and "Casad East" had a closure of 01".68 and was adjusted by least square method holding the two tellurometer lengths fixed. The angles from the three stations to the common azimuth mark were similarly adjusted. The resulting position of "Casad" and "Casad East" will be within 0.4 meter in relation to the second-order control in the area. Computations were performed on the Clark 1866 Spheroid, 1927 North American Datum.

The elevations of stations "Casad", "Casad West" and "Casad East" were determined by 4th order leveling from USC&GS First Order BM K-15. The loop closure was 0.10 ft. The elevation of each of the three stations established will be within 1 ft. in relation to the first order USC&GS level line.

- 1 -

Enclosure 1

The Master Description and Position cards, computations and all field records are retained in the Americas Division.

APPROVED: W. L. Norris
W. L. NORRIS

Frank E. Williams
FRANK E. WILLIAMS
Geodesist
Americas Division
Dept. of Geodesy

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Enclosure 1

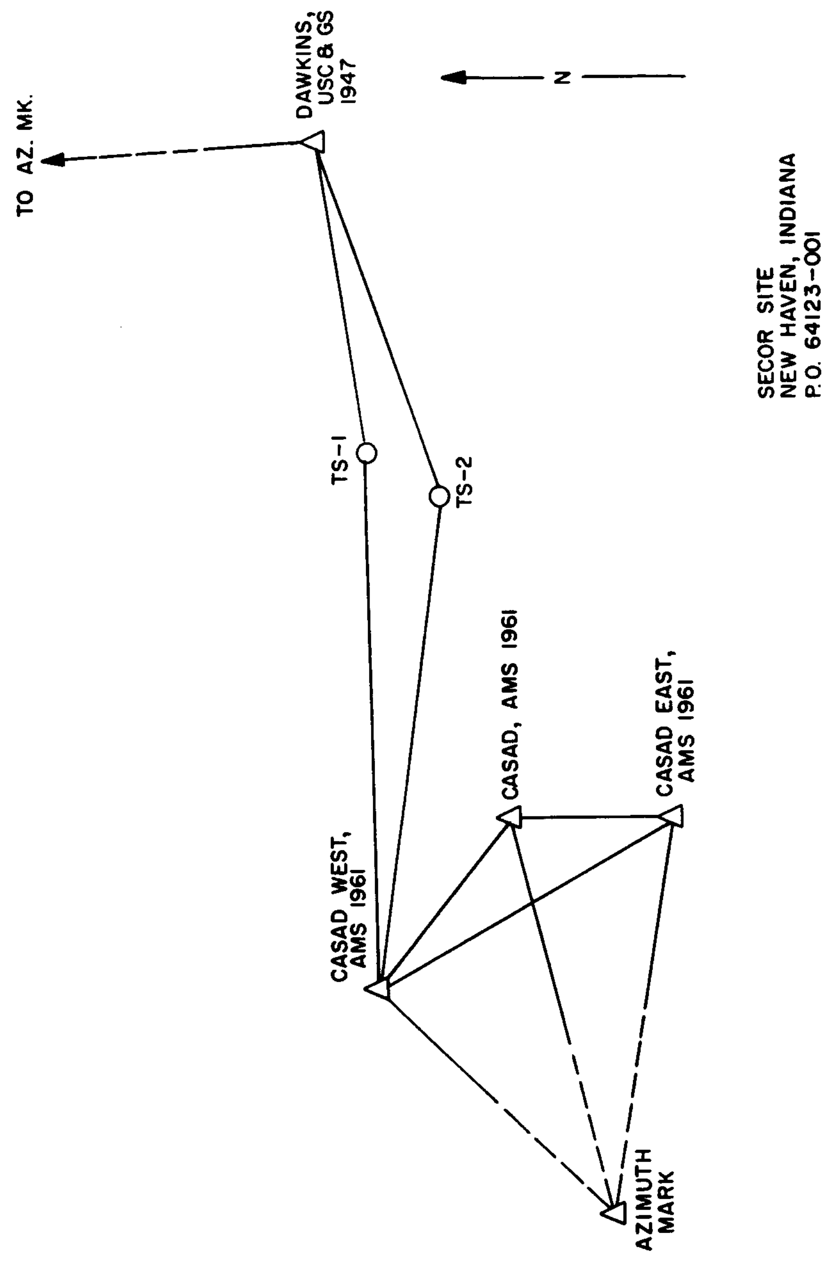

Fig. A1. Secor site

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Enclosure 1

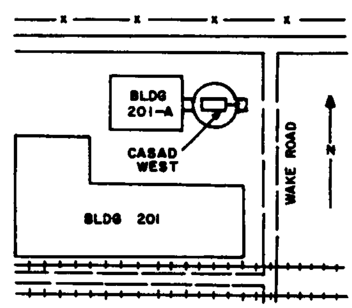
DUPLICATE MASTER COPY

COUNTRY	TYPE OF MARK	STATION	
United States	Disk	CASAD WEST, AMS 1961	
LOCALITY: New Haven, Indiana	STAMPING ON MARK: CASAD WEST AMS 1961	AGENCY (CAST IN MARK): C. of E.	ELEVATION: 816.87 FEET / 248.98 METERS
LATITUDE: 41° 04' 43"746	LONGITUDE: 84° 56' 35"792	DATUM: 1927 NAD	ORDER: 4th LL
NORTHING: 4,549,332.83	EASTING: 672,777.25	GRID AND ZONE: UTM 16	DATUM: Sea Level Datum of 1929
NORTHING:	EASTING:	GRID AND ZONE:	ESTABLISHED BY AGENCY-DATE: AMS 1961

TO OBTAIN Universal Transverse Mercator GRID AZIMUTH, ADD 178 38 54 TO THE GEODETIC AZIMUTH

OBJECT	(GEODETIC) AZIMUTH	BACK AZIMUTH	GEOD. DISTANCE (M)	GRID DISTANCE (M)
Az. Mk. (Water Tank)	58° 52' 08"0			
Casad	276 21 35.9	96° 21' 56"0	718.605	718.583
Casad East	279 19 48.7	99 20 12.2	846.260	846.232

The mark is a Corp of Engineer disk set in the top of a concrete tower flush with the top of the tower, and is stamped: "CASAD WEST AMS 1961".



To reach the station from the intersection of U.S. Highway 24 and State Highway 14 in New Haven, proceed east on State Highway 14, 3.35 miles to the main gate of Casad Engineer Depot; turn left (north) and go thru the main gate 0.1 mile to Plantation Road; turn right (east) and go 0.1 mile to Delaware Avenue; turn left (north) on Delaware Avenue and go 0.3 mile to Virgin Road; turn right (east) on Virgin Road and go 0.3 mile to Wake Road; turn left (north) on Wake Road and go 0.1 mile to coal tower and station.

The mark is located approximately 4 miles (airline) east of New Haven, Indiana, approximately 0.5 miles (airline) north of State Highway 14, 3.2 feet south of south edge of the coal elevator shute, 3.8 feet north of the south edge of the top of the coal tower and 4.5 feet southeast of the southeast corner of a trap door.

Azimuth Mark: The center of the top of the Casad Engineer Depot water tank. The point observed is the center of the light standard on the top of the tank.

GEOGRAPHIC POSITION
INTERNATIONAL SPHEROID TANGENT AT
MEADES RANCH.
LAT. 41° 04' 43."496
LONG. 84° 56' 36."752

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Enclosure 1

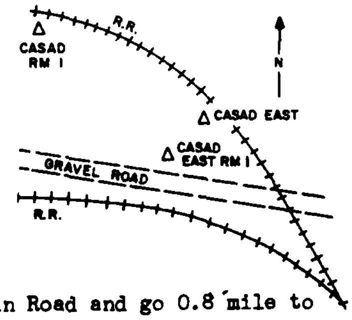
DUPLICATE

COUNTRY United States	TYPE OF MARK Disk	STATION CASAD EAST, AMS 1961	
LOCALITY New Haven, Indiana	STAMPING ON MARK CASAD EAST AMS 1961	AGENCY (CAST IN MARK) C. of E.	ELEVATION 764.12 FEET 232.90 METER
LATITUDE 41° 04' 39".297	LONGITUDE 84° 56' 00".020	DATUM 1927 NAD	ORDER 4th LL
NORTHING 4,549,215.38	EASTING 673,615.29	GRID AND ZONE UTM 16	DATUM Sea Level Datum of 1929
NORTHING	EASTING	GRID AND ZONE	ESTABLISHED BY AGENCY DATE AMS 1961

TO OBTAIN Universal Transverse Mercator GRID AZIMUTH ADD 178 38 30 TO THE GEODETIC AZIMUTH

OBJECT	(GEODETIC) AZIMUTH	BACK AZIMUTH	GEOD DISTANCE (M)	GRID DISTANCE (M)
RM # 1 (Casad East)	27° 35' 04".1		10.49	10.49
Az. Mk.	78 56 42.1			
Casad West	99 20 12.2	279° 19' 48".7	846.260	846.232
Casad	115 28 57.2	295 28 53.8	133.895	133.891
RM #1 (Casad)	115 42 38.6	-	66.991	66.959

The mark is a Corp of Engineer disk set in the top of a concrete monument set flush with the ground and is stamped: CASAD EAST AMS 1961.



To reach from the intersection of U.S. Highway 24 and State Highway 14 in New Haven, proceed east on State Highway 14, 3.35 miles to the main gate of Casad Engineer Depot; turn left (north) and go thru the main gate 0.1 mile to Plantation Road; turn right (east) and go 0.1 mile to Delaware Avenue; turn left (north) on Delaware Avenue and go 0.3 mile to Virgin Road; turn right (east) on Virgin Road and go 0.8 mile to station site and station.

The mark is located approximately 4 miles (airline) east of New Haven, Indiana, approximately 0.4 miles (airline) north of State Highway 14, 219.86 feet east-south-east of Casad RM 1, 138.7 feet northwest of the centerline of intersection of gravel road and railroad, 108.5 feet north of the north rail of a railroad, 69.0 feet north of the centerline of a gravel road, 48.8 feet southwest of the southwest rail of a railroad and 8.2 feet west of a 4x4 inch witness post.

Reference Mark 1 is a Corp of Engineer disk set in the top of a concrete monument set flush with the ground, and is stamped: CASAD EAST RM 1 AMS 1961. It is 34.42 feet south-southwest of station Casad East, 78.8 feet southwest of the southwest rail of a railroad, 36.2 feet north of the centerline of a gravel road and 2.6 feet north of a 4x4 inch witness post.

The Azimuth Mark is the center of the top of the Casad Engineer Depot water tank. The point observed is the center of the light standard on the top of the tank.

GEOGRAPHIC POSITION
INTERNATIONAL SPHEROID TANGENT AT
MEADES RANCH
LAT. 41° 04' 39".047
LONG. 84° 56' 00".980

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Enclosure 1

DUPLICATE MASTER COPY

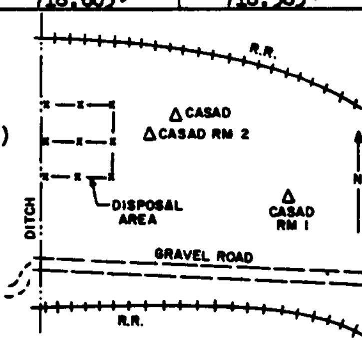
COUNTRY	TYPE OF MARK	STATION	
United States	Disk	Casad, AMS 1961	
LOCALITY New Haven, Indiana	STAMPING ON MARK CASAD AMS 1961	AGENCY (CAST IN MARK) C. of E.	ELEVATION 763.72 FEET 232.78 METERS
LATITUDE 41° 04' 41".164	LONGITUDE 84° 56' 05".198	DATUM 1927 NAD	ORDER 4th LL
NORTHING 4,549,270.09	EASTING 673,493.09	GRID AND ZONE UTM 16	DATUM Sea Level Datum of 1929
NORTHING	EASTING	GRID AND ZONE	ESTABLISHED BY-AGENCY-DATE AMS 1961

TO OBTAIN Universal Transverse Mercator GRID AZIMUTH ADD 178 38 33 TO THE GEODETIC AZIMUTH

OBJECT	(GEODETIC) AZIMUTH	BACK AZIMUTH	GEOD DISTANCE (M)	GRID DISTANCE (M)
RM #2	35° 49' 42".0		11.354	11.354
Az. Mk.	75 28 36.5		—	—
RM #1	295 15 20.1		66.988	66.986
Casad East	295 28 53.8	115° 28' 57".2	133.895	133.891
Casad West	96 21 56.0	276 21 35.9	718.605	718.583

The mark is a Corp of Engineer disk set in the top of a concrete monument set flush with the ground and is stamped "CASAD, AMS 1961".

The mark is located approximately 4 miles (airline) east of New Haven, Indiana, approximately 0.4 mile north of State Highway 14; 266.7 feet north of north rail of the railroad, 212 feet north of the centerline of a gravel road, 131.6 feet east of a fence corner, 107.7 feet south of south rail of railroad and 8.3 feet east-southeast of a 4x4 witness post.



Reference Mark 1: A Corp of Engineer disk set in the top of a concrete monument set flush with the ground and is stamped: CASAD RM 1 AMS 1961. It is 219.86 feet east-southeast of station CASAD, 167.5 feet north of the north rail of a railroad, 140.3 feet north of the centerline of a gravel road, 126.0 feet south-southwest of the south rail of a railroad and 2.7 feet south of a 4x4 inch witness post.

Reference Mark 2: A Corp of Engineer disk set in the top of a concrete monument set flush with the ground and is stamped: CASAD RM 2 AMS 1961. It is 37.25 feet south-southwest of station Casad, 237.3 feet north of the north rail of a railroad, 185.3 feet north of the centerline of a gravel road, 140.3 feet south of the south rail of a railroad, 113.8 feet southeast of a fence corner and 3.2 feet east of a 4x4 inch witness post.

Azimuth Mark: The center of the top of the Casad Engineer Depot water tank. The point observed on is the center of the light standard on the top of the tank.

To reach from the intersection of U.S. Highway 24 and State Highway 14 in New Haven: Proceed east on State Highway 14 3.35 miles to the main gate of Casad Engineer Depot; turn left (north) and go thru the main gate 0.1 mile to Plantation Road; turn right (east) and go 0.1 mile to Delaware Avenue; turn left (north) on Delaware Avenue and go 0.3 mile to Virgin Road; turn right (east) on Virgin Road and go 0.7 mile to station site and station.

GEOGRAPHIC POSITION
INTERNATIONAL SPHEROID TANGENT AT
MEADES RANCH.
LAT. 41°04'40".914
LONG. 84°56'06".159

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Enclosure 2* W. M. Kaula

RECOMMENDED EARTH GRAVITATIONAL AND GEOMETRICAL CONSTANTS FOR TRACKING AND ORBIT COMPUTATION

1. Gravitational

a. Numerical values

In the formula

$$U = (GM/r) \quad 1 - \Sigma_n J_n \, (a_e/r)^n \, P_n(\sin\phi)$$

where $P_n(\sin\phi)$ is the Legendre polynomial and ϕ is the geocentric latitude:

$$GM = 3.986032\ (\pm 0.000030) \times 10^{20} \mathrm{cm}^3/\mathrm{sec}^2$$

$$J_2 = 1082.30\ (\pm 0.2) \times 10^{-6}$$

$$J_3 = -2.3\ (\pm 0.1) \times 10^{-6}$$

$$J_4 = -1.8\ (\pm 0.2) \times 10^{-6}$$

$$J_n = 0.0\ (\pm < 1.0) \times 10^{-6},\ n \geq 5$$

$$J_{nm} = 0.0\ (\pm < 2.0) \times 10^{-6},\ m \neq 0.$$

$$a_e = 6.378165\ (\pm 0.000025) \times 10^8 \mathrm{cm}$$

b. Remarks

The values of GM, J_2 and a_e are consistent with the values of geodetic parameters.

$$f = 1/298.30$$

$$g_e = 978.030\ \mathrm{cm/sec}^2$$

The values of a_e, f, g_e are those specified in the DOD World Geodetic System 1960 and are here recommended for sake of consistency. In addition they are close to the best estimates for these parameters. Reasonable alternative values based on terrestrial geodetic data (e.g., those in NASA Tech. Note D-702) differ by less than 20 meters in a_e, .001 cm/sec^2 in g_e, and 0.1 in 1/f.

The value of g_e incorporates a correction of -.013 cm/sec^2 to the Potsdam standard absolute gravity.

*Enclosure B to Minutes of the NASA Earth Model Meeting, May 4, 1961.

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Enclosure 2

The values of J_3 and J_4 are compromises between the values obtained by the principal investigators of satellite orbits (1), (2), (3), with greatest weight to Kozai (3), and the given uncertainties are based on the discrepancies between these results. The values of J_2 by these same investigators range from 1082.19 to 1082.79 x 10^{-6}. The magnitude of effect of the omitted J_{nm} on satellite positions is about ±400m. or less (4).

The most serious discrepancy in determination of gravitational parameters is between the GM from terrestrial data, 3.986032 x 10^{20} cgs., and that based on the lunar mean motion and the radar measurement of the Moon's distance: 3.986141 (±0.000040) x 10^{20} cgs. This value depends on the Moon/Earth mass ratio of 1/81.375 of Rabe (5); 3.986048 is obtained from Delano's 1/81.219 (6). However, the stated uncertainty depends mainly on the uncertainties in the radar measurement and the lunar radius.

2. Geometrical

a. Numerical values:

For stations connected to the principal geodetic control system of the Western Hemisphere (see Fig. B1), add the corrections listed to rectangular coordinates in the (u, v, w) system (axes toward Lat., Long.: (0°, 0°), (0°, 90°E), (90°N), respectively) on the stated conventional datums:

	Meters		
	Δu	Δv	Δw
NASA TN 702 - NAD	-23	+142	+196
NASA TN 702 - SAD	-303	+98	-315
NASA TN 702 - SAO SP 59	+4	+299	+15
NASA TN 702 - Vanguard	-12	+235	+120
σ (NASA TN 702)	±26	±22	±22

For stations connected to the Europe-Africa-Siberia-India geodetic system:

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Enclosure 2

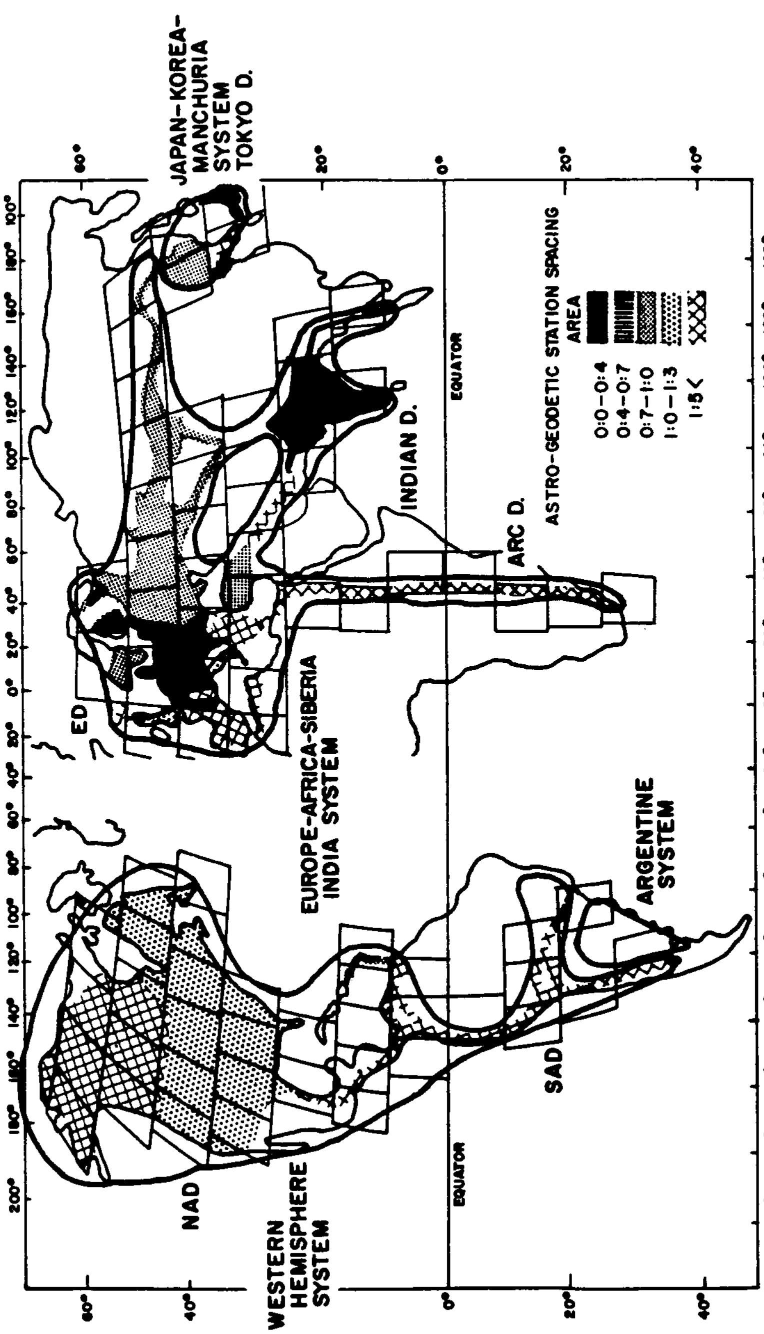

Fig. B1. Astrogeodetic geoid data station spacing and distribution

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	Meters		
	Δu	Δv	Δw
NASA TN 702 - ED	-57	-37	-96
NASA TN 702 - Indian	+200	+782	+271
NASA TN 702 - Arc	-109	-70	-289
NASA TN 702 - SAO SP 59	-150	-2	-33
σ (NASA TN 702)	±23	±29	±23

For stations connected to the Japan-Korea-Manchuria geodetic system:

	Meters		
	Δu	Δv	Δw
NASA TN 702 - Tokyo	-89	+551	+710
NASA TN 702 - SAO SP 59	-29	-209	-147
σ (NASA TN 702)	±40	±53	±40

For stations connected to the Australian system:

	Δu	Δv	Δw
NASA TN 702 - Sydney	+198	+262	-21
NASA TN 702 - SAO SP 59	+149	-83	+116
Estimated σ (NASA TN 702)	±75	±90	±35

For stations connected to the Argentine system:

	Δu	Δv	Δw
NASA TN 702 - SAO SP 59	-81	+131	+105
(Argentine Datum - SAO SP 59)			
Estimated σ (NASA TN 702)	±180	±160	±160

For stations not connected to any of the above principal geodetic systems, but which have an astronomical position or which are connected to a local system, the geodetic latitude and longitude must be assumed to be that of the astronomic position or local system, and the u, v, w, coordinates obtained by:

$$u = (\nu + h) \cos\phi \cos\lambda$$

$$v = (\nu + h) \cos\phi_g \sin\lambda$$

$$w = \left[(1 - e^2)\, \nu + h\right] \sin\phi_g$$

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Enclosure 2

Where $\nu = a_e (1 - e^2 \sin\phi_g)^{-1/2}$, $e = 2f - f^2$, and h is the elevation above the ellipsoid. If a_e = 6378165 m. and f = 1/298.30 are used, and the height above the ellipsoid assumed to be identical with height above sea level, then the standard error of position in the radial direction should be:

$$\sigma(r) = \pm 45 \text{ meters}$$

If the geoid heights from NASA Tech Note 702 Fig. 2 or 3 is added to the height above sea level, then there will be a slight improvement to about ±35 meters.

For the horizontal coordinates at a station in a geophysically stable continental area:

$$\sigma(r\phi) = \sigma(r\lambda \cos\phi) \approx \pm 170 \text{ meters}$$

For the horizontal coordinates from a single astronomic position on an island or in a geophysically disturbed area (mountains, etc.):

$$\sigma(r\phi) = \sigma(r\lambda \cos\phi) \approx \pm 350 \text{ meters}$$

By using the mean position obtained by connecting astronomic observations on opposite sides of an island by traverse this may be improved to about:

$$\sigma(r\phi) = \sigma(r\lambda \cos\phi) \approx \pm 250 \text{ meters}$$

By using topographic isostatic corrections of the deflections of the vertical this may be further improved to about:

for a single station: $\sigma(r\phi) = \sigma(r\lambda \cos\phi) \approx 200$ meters

for the mean from observations on opposite sides:

$$\sigma(r) = \sigma(r\lambda \cos\phi) \approx \pm 120 \text{ meters}$$

Enclosure 2

b. Remarks

The world geodetic system identified by NASA TN 702 herein is, for the principal Western Hemisphere, Europe, Siberia, Africa, India, and Japan-Korea-Manchuria systems, that obtained in the same general solution as the ellipsoid parameters, gravitational coefficients, etc., described in NASA Tech Note D 702 (7). This solution is recommended because it employs the most elaborate techniques and utilizes by far the most data of all solutions based solely on unclassified data. The values recommended for Argentina and Australia are based on the assumption of tangency at the geodetic datum "origins" of an $a_e = 6378165 + N_o$, 1/298.3 ellipsoid, where N_o is the geoid height at the datum origin given in Figs. 2 and 3 of NASA Tech Note D 702.

The Vanguard Datum was based on the assumption of tangency to NAD at its origin (97°N, 263°E) of the Hough Ellipsoid:

$$a_e = 6378270, \qquad f = 1/297.0$$

The SAO SP 59 datum (8) is based on the assumption of tangency to the conventional datums, corrected by gravimetrically computed deflections of the vertical (except in Argentina), of the International Ellipsoid:

$$a_e = 6378388, \qquad f = 1/297.0$$

The large differences from NASA TN 702 datum are due mainly to this use of an obsolete ellipsoid, secondarily to the utilization of much less observational data.

Note that all datum shifts are described as translations: there are no rotations. For properly observed geodetic systems the orientation error is negligible. Orientation of geodetic systems is obtained from the stars through "Laplace stations", at which astronomic azimuth and longitude are observed.

The standard error for difference of position between two stations connected to the same geodetic control system should always be less than ±20 meters.

The standard errors for astronomic positions in a continental area are based on autocovariance analysis of gravimetry (9).

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Enclosure 2

The standard error for astronomic positions on islands is based on a sample of 69 islands in the West Indies connected to the continental geodetic system by Hiran trilateration.

REFERENCES

(1) O'Keefe, J. A., Eckels, A. and Squires, R. K., The gravitational field of the earth. Astron J. 66, 245-253, 1959.

(2) King-Hele, D. G., The earth's gravitational potential deduced from the orbits of artificial satellites. Geophys. J. 4, in press, 1961.

(3) Kozai, Y., The gravitational field of the earth derived from motions of three satellites. Astron J. 66, 8-10, 1961.

(4) Kaula, W. M., Analysis of gravitational and geometric aspects of geodetic utilization of satellites. NASA Tech Note D-572, 38 pp., and Geophys. J. 4, in press, 1961.

(5) Rabe, E., Derivation of fundamental astronomical constants from the observations of Eros during 1926-1945. Astron. J. 55, 112-126, 1950.

(6) Delano, E., The lunar equation from observations of Eros, 1930-31. Astron J. 55, 129-133, 1950.

(7) Kaula, W. M., A geoid and world geodetic system based on a combination of gravimetric, astrogeodetic, and satellite data. NASA Tech Note D-702, and J. Geophys. Res. 66 6, in press, 1961.

(8) Veis, G., The positions of the Baker-Nunn camera stations Smithsonian Astro. Obs. Spec. Rep. 59, 5 pp, 1961.

(9) Kaula, W. M., Statistical and Harmonic Analysis of Gravity. J. Geophys. Res. 64, 2401-2421, 1959.

(10) Veis, G., Geodetic uses of artificial satellites. Smithsonian Inst. Astrophys. Obs. Contrb. to Astrophys. 3, 95-161, 1960.

(11) Kaula, W. M., Celestial Geodesy Adv. in Geophysics 9, in press, 1961.

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